# Profit Maximization Slides

**Econ 360**

Summer 2025

# Learning Outcomes

- Define a firm's profit function in both the short run and the long run.
- Algebraically and graphically determine a firm's profit maximizing output and demand for inputs.
- Describe the conditions a firm satisfies when it achieves profit maximization in terms of input prices and marginal products.
- Algebraically and graphically determine a firm's returns to scale and whether profit maximization makes sense given a firm's production function.

# Where We Are

- We know about a firm's production function, and we can draw isoquants for a given firm based on their production function.
- We have not yet used that isoquant to figure out what quantity a firm will pick.
- We will assume that all a firm cares about is maximizing their profit.
- We will also assume a firm is in a competitive market, and therefore cannot choose the price they pay for their inputs nor the price they can charge for their product.
  - The firm is a **price taker** in both the input and the final product market.

# Notation Reminder: Technology

- Firms turn **inputs** into **output**.
  - We use y's to denote outputs. I.e. $\{y_1, y_2, ..., y_n\}$.
  - These outputs have prices $\{p_1, p_2, ..., p_n\}$.
  - We use x's to denote inputs. I.e. $\{x_1, x_2, ..., x_m\}$.
  - These inputs have prices $\{w_1, w_2, ..., w_m\}$.
- In this class, we will mostly focus on one output $y$.
- We will also generally focus on two main inputs, **L**abor, or workers, and **K**apital, or machines/buildings/land/any non-labor input.
  - Econ decided that K should be for capital since we use C for costs generally.
  - We will use $w$ for the price of Labor and $r$ for the price of K/Capital.

# Profit Maximization Generally

- If a firm sells $q$ units for price $p$ then **Total Revenue**=$p \cdot q$.

- If a firm hires $L$ workers for $w$ per worker and $K$ machines for $r$ per machine, the firm's **Total Cost**=$w \cdot L + r \cdot K$.

- Therefore the firm's **Profit**=$\pi = p \cdot q - w \cdot L - r \cdot K$.

- For the more general case we would still have Total Revenue-Total Cost, or

$$\pi = \Sigma_{i=1}^{n} p_i \cdot y_i - \Sigma_{j=1}^{m} w_j \cdot x_j$$

# Profit Maximization

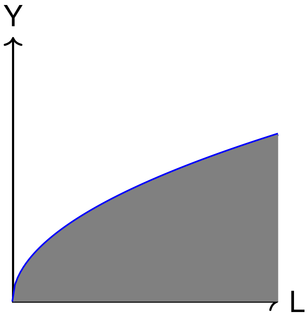
- With any optimization, we have an **objective** and a **constraint.**
  - In utility maximization, our objective was our utility function and our constraint was our budget constraint.
- Here, our constraint is how much we are able to produce.
  - Our constraint is our **production function**.
- Our objective is to maximize profit.

- So our fundamental goal is to

**Maximize profit subject to our production function!**

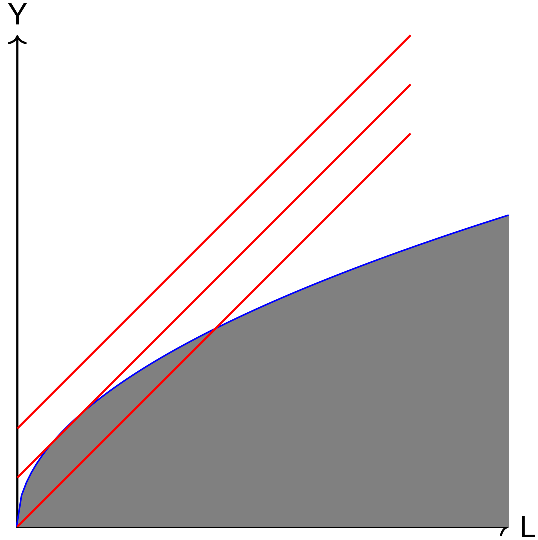
# Profit Maximization in the Short Run

- To start, let's use our 2-input 1-output example in the short run.
- That way, K is fixed. So really our production function is $y = f(L, \bar{K})$.
- We can use the production function we used in the last set of slides.



- To this graph, we can add **isoprofit** lines.
- Basically, all combinations of y and L that give us the same level of profit.
- As I increase y, my revenue increases, but I also increase L, so my costs increase. Therefore my profit stays constant.
- There are an infinite amount of these isoprofit lines, one for each level of profit.
- I will draw a couple in red.

# Isoprofit Lines



# Profit Maximization

- With this picture in mind, how do we solve profit maximization?
- We simply figure out where our objective (our isoprofit line) is tangent to our constraint (our production function)!
- Just like we did before in utility maximization.
- Alternatively, we can also use marginal cost=marginal benefit. Again, just like in utility maximization.
- Let's talk about the point of tangency first.

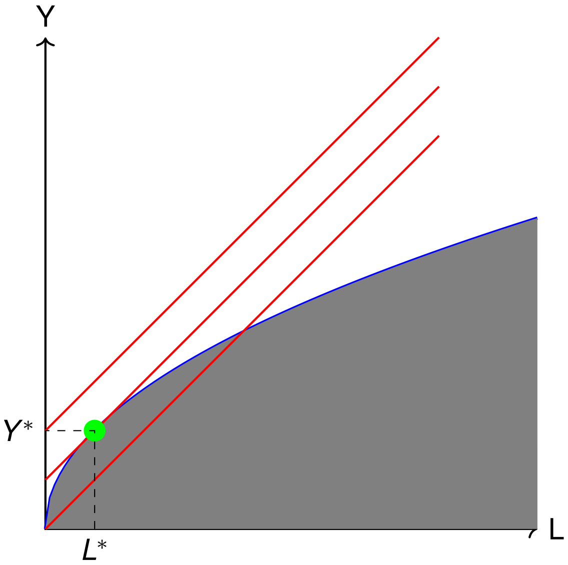
# Profit Maximization–Tangency

- With only 1 input, the slope of the production function is the marginal product of L, or the derivative of the production function with respect to L.
- The slope of the isoprofit line is just ratio of the output price compared to the price of L, or w.
  - Increasing y by one increases profit by $p$.
  - In order to stay at the same profit costs must also increase by $p$.
  - Therefore we need to increase the amount of L such that costs increase by $p$.
  - That amount of Labor we need is $\frac{w}{p}$.
- Therefore the tangency point here is $MP_L = \frac{w}{p}$.

# Profit Maximization–MB=MC

- We can also use marginal benefit=marginal cost to get the same equation as in the previous slide.
- Here we are thinking about the marginal benefit and cost of hiring another worker, or increasing L.
- The marginal cost of hiring an additional worker is $w$ or the wage.
- The benefit of hiring an additional worker is the **value** of the marginal product, or $p \cdot MP_L$.
  - For example, if hiring an additional worker increases output by 2. ($MP_L = 2$)
  - Suppose the price per unit of output is \$4.
  - Then the marginal benefit is the 2 units times \$4 which is $p \cdot MP_L = \$8$.
- Then we have $p \cdot MP_L = w \implies MP_L = \frac{w}{p}$.

# Profit Maximization–Graphed



# Profit Maximization–2 Inputs

- If we think about the long run, the firm can choose both K and L.
- But the same marginal benefit=marginal cost idea still holds, just for both inputs.
- We can simply use those two equations plus the production function and profit function to figure out how much K and L the firm should use!

$$MB_L = p \cdot MP_L = w = MC_L$$
$$MB_K = p \cdot MP_K = r = MC_K$$

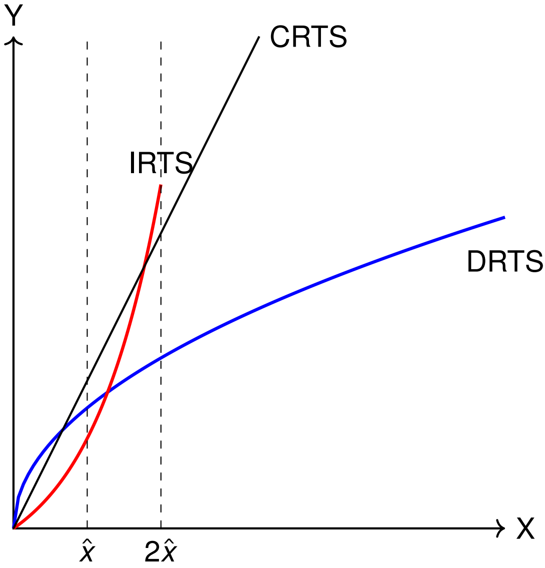
# When does Profit Maximization Work?

- By "work" we mean we can find a finite amount of L and K that is profit maximizing for the firm given the firm's production function.
- For example, if our answer is that the profit maximizing amount of L and K is infinity, that is not really an answer to the profit maximization problem.
- Similarly, if our answer is the firm can choose any amount of L and K they want and be profit maximizing, that is also not really an answer.
- This comes down to our production functions from earlier and returns to scale.
- **The basic idea is that we need the production function to exhibit decreasing returns to scale.**

# Returns to Scale–From Technology Slides

- Earlier we talked about how it made sense that as we increase the amount of one input, holding the amount of the other inputs constant, output should increase at a decreasing rate.
- That was an assumption we made and something that is not always true.
- **Question**: How can we determine if our assumption holds for a given production function or not?
  1. If the marginal products are diminishing for all inputs, that would tell us our assumption holds.
  2. An even easier way is to ask whether doubling our inputs results in a level of output that is less than double, double, or more than double.

# Returns to Scale-Graphed From Technology Slides



# Mathematically Determining RTS

- Suppose we have a production function $y = f(K, L)$.
- We can figure out what we could get if we plug in $f(2K, 2L)$ into our production function.
  - This is the doubling of our inputs.
- We can then double our output, or calculate $2y$.
- We then figure our returns to scale according to the following rule:
  - IRTS: $2y < f(2K, 2L)$.
  - CRTS: $2y = f(2K, 2L)$.
  - DRTS: $2y > f(2K, 2L)$.